Anomalous kinetics of the annihilation process in the turbulent environment

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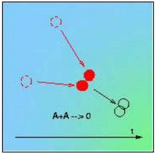
One-species reaction model

- broad class of chemical reactions $A + A \xrightarrow{\lambda_0} \varnothing$



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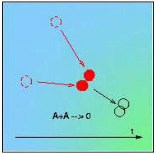

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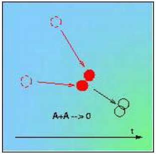

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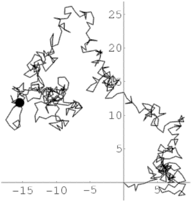

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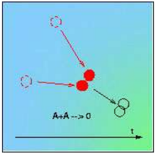

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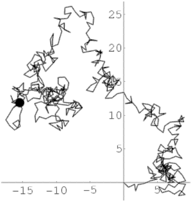

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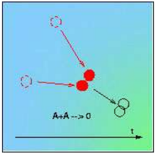

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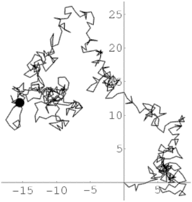

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- What's the value of decaying exponent α, $n(t) \xrightarrow{t\to\infty} t^{-\alpha}$?

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- for $d > 2$ $\quad V(t) \sim t \Rightarrow n(t) \sim t^{-1}$

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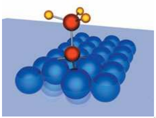
  - catalysis on surface of some cells



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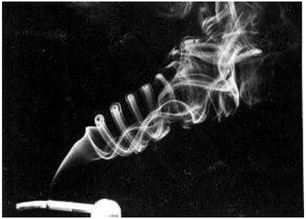

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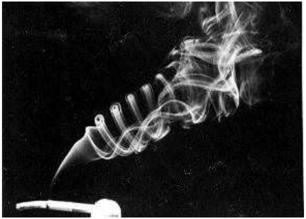

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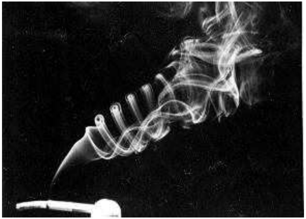

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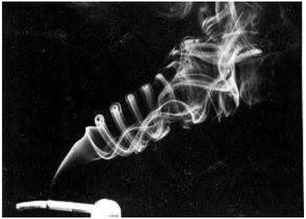

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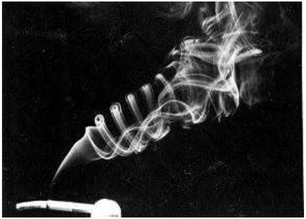

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- probabilistic description by statistical functions - probability density functions, correlation and structure functions etc.

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- correlator in Fourier representation $d_f(k) = g_{10}\nu^3 k^{4-d-2\epsilon} + g_{20}\nu^3 k^2$

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- $\Rightarrow$ suggestion of using second quantization method

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- discrete model (particles on the lattics) $\xRightarrow{\text{Doi formalism}}$ continuous model

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- from the actions, (2) and (11) propagators and interaction vertex could be extracted

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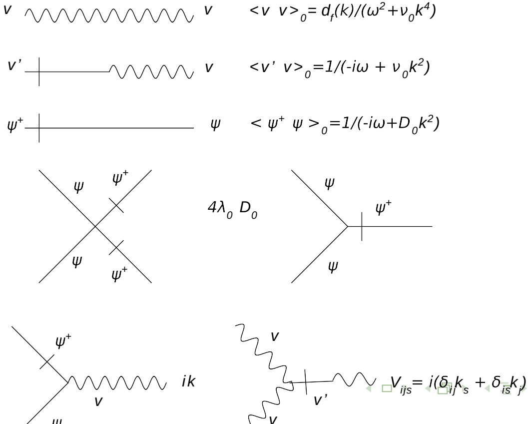
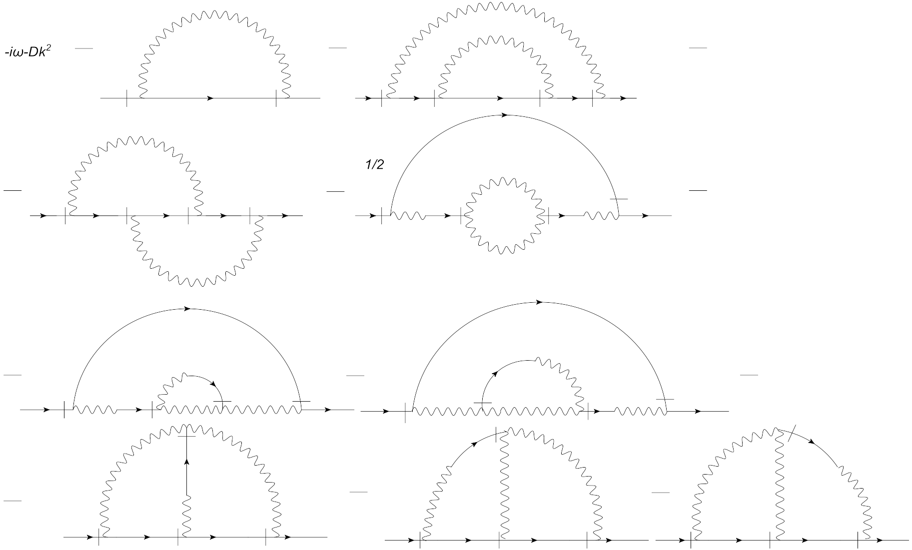
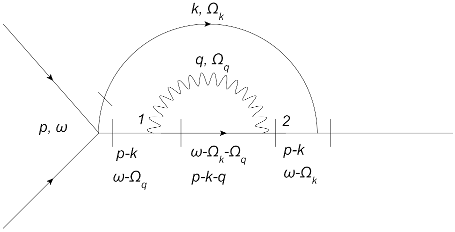
- diagrammatic representation

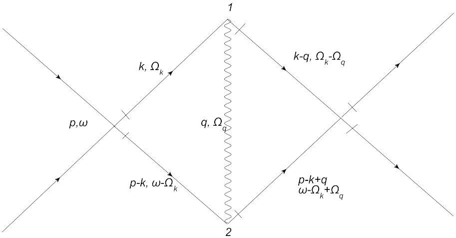


Dyson equation for propagator $\Delta_{\psi^+\psi} = -\Gamma_{\psi^+\psi}$



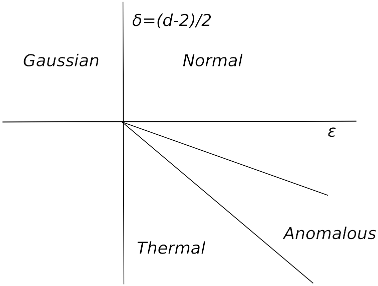
Two-loop contribution to Z_4





Results in one-loop order

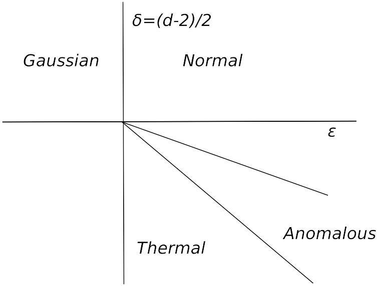
decaying exponent of the particle concentration $n(t) \propto t^{-\alpha}$



Fixed point	α	region of stability

Results in one-loop order

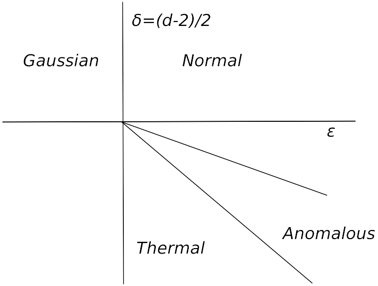
decaying exponent of the particle concentration $n(t) \propto t^{-\alpha}$



Fixed point	α	region of stability
Gaussian	1	$\epsilon < 0, \delta > 0$

Results in one-loop order

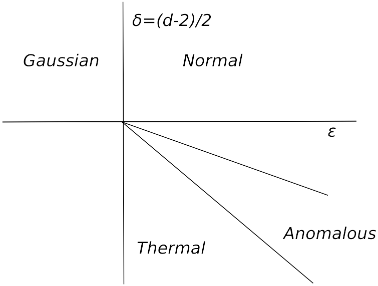
decaying exponent of the particle concentration $n(t) \propto t^{-\alpha}$



Fixed point	α	region of stability
Gaussian	1	$\epsilon < 0, \delta > 0$
Thermal	$1 + \frac{\delta}{2}$	$\delta < 0, 2\epsilon + 3\delta < 0$

Results in one-loop order

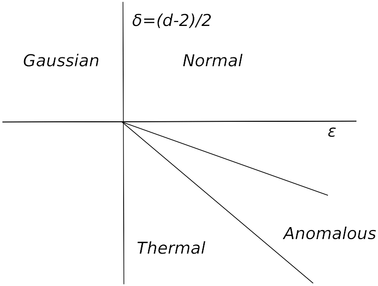
decaying exponent of the particle concentration $n(t) \propto t^{-\alpha}$



Fixed point	α	region of stability
Gaussian	1	$\epsilon < 0, \delta > 0$
Thermal	$1 + \frac{\delta}{2}$	$\delta < 0, 2\epsilon + 3\delta < 0$
Anomalous kinetics	$\frac{1+\delta}{1-\epsilon/3}$	$\epsilon > 0, -2\epsilon/3 < \delta < -\epsilon/3$

Results in one-loop order

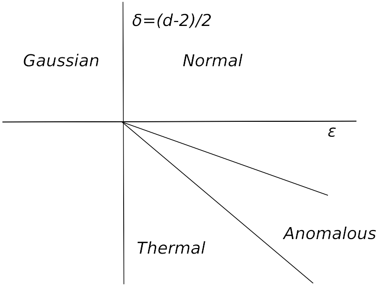
decaying exponent of the particle concentration $n(t) \propto t^{-\alpha}$



Fixed point	α	region of stability
Gaussian	1	$\epsilon < 0, \delta > 0$
Thermal	$1 + \frac{\delta}{2}$	$\delta < 0, 2\epsilon + 3\delta < 0$
Anomalous kinetics	$\frac{1+\delta}{1-\epsilon/3}$	$\epsilon > 0, -2\epsilon/3 < \delta < -\epsilon/3$
Normal kinetics	1	$\epsilon > 0, \delta > -\epsilon/3$

Results in one-loop order

decaying exponent of the particle concentration $n(t) \propto t^{-\alpha}$



Fixed point	α	region of stability
Gaussian	1	$\epsilon < 0, \delta > 0$
Thermal	$1 + \frac{\delta}{2}$	$\delta < 0, 2\epsilon + 3\delta < 0$
Anomalous kinetics	$\frac{1+\delta}{1-\epsilon/3}$	$\epsilon > 0, -2\epsilon/3 < \delta < -\epsilon/3$
Normal kinetics	1	$\epsilon > 0, \delta > -\epsilon/3$
Driftless	$1 + \delta$	unstable

Results and plan for future work

- achieved results in one-loop order
- completed calculation of the RG functions to the two-loop order
- use rapid-change model for the description of advecting field
- sink and sources of particles

Ďakujem za pozornosť

Thank you for your attention

спасибо за внимание